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Title:  
**MEASURING THE QUALITY OF GUITAR TONE**

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Summary:

A method for the determination of the tone quality of a classical guitar is described. It was applied to several high and low quality classical guitars. In comparison to bad tones, the timbre of good tones consists of stronger consonant (pleasant) and weaker dissonant (unpleasant) intervals. This empirical criterion of tone quality was named the 'rule of consonance-dissonance' (RC-D). RC-D was defined mathematically, and interpreted in physical and musical terms. RC-D allows a luthier to pursue systematically the tone quality during guitar production and to improve the instrument's tone after its assembly.

## 1. INTRODUCTION

Guitarmakers aim at achieving high tone quality of their instruments. However, in the literature not much has been said about the way tone quality should be measured and determined. For example, Cumpiano and Natelson [1] wrote about good tone of a guitar but they did not define the tone quality in physical terms. Richardson [2] discussed the important role of the low-order modes of vibration of the stringed musical instrument. These modes are responsible for much of the sound radiation at both low and high frequencies. Because low (high) frequency normal modes radiate only at low (high) frequencies this statement indicates that there is some correlation between the sound radiation of an instrument in low and high frequency range. It is logical that this correlation depends on (i) the design features, and (ii) the material of an instrument. Thus, the aim of our research was to answer what is the most important feature of a tone of a good guitar and how it can be distinguished from a bad tone. The basis for evaluation of our results was a definition of the consonance and dissonance between two tones by Olson [3] and partially by Sethares [4]. Two guitar samples were examined, consisting of four good and four extremely bad guitars, respectively. The paper describes the sound measurements of these two guitar samples and the analysis of tone quality differences. In addition, the frequency response function of a guitar which depends on mechanical properties of both top and back plates was related to its tonal quality and to the statement about importance of low-order

modes (see above). On the basis of the results a rule for determining guitar tone quality has been proposed, which we hope satisfies the criterion of objectivity.

## 2. METHODS

### 2.1. DEFINITIONS

All measurements were performed in the same ordinary room without special sound insulation. This was thought to be sufficient because only the difference between the good and bad guitar tones was measured. The surrounding noise was at least 15 dB (0 dB  $\equiv$  0.00002 Pa) below the sound pressure level (*i.e.*, SPL) of any frequency of interest. The room temperature was ranging from 18 to 22 °C and relative humidity from 40 to 55%. The differences in quality of guitar tones (see below) were noticeable during the whole experimentation, thus relatively large deviation in the temperature and humidity was not significant in this case.

The following characteristics were judged subjectively by ear: The richness of the timbre of the tones of good guitars in comparison to the bad guitars was obvious. The duration of tones was longer for the good guitars. The good guitars enabled quality playing of quiet tones as well as loud tones, which was not the case for the bad guitars with bad dynamic capability. The buzz tone for bad guitars was relatively frequent. The Wolf tone (undesired pulsation of sound intensity) [5] occurred for high tones for both tested groups of guitars.

### 2.2. SUBJECT OF MEASUREMENTS

Three tones, "F" (fundamental frequency of 87.3 Hz, 6th string - 1st fret), "B" (123.5 Hz, 5th string - 2nd fret), and "g" (196.0 Hz, 3rd string), were subjected to analysis. The selection of these tones was guided by the properties of the A/D converter in the data acquisition board, which enabled only certain sample rates.

The plucked string vibrates with fundamental frequency and higher harmonic components, which are multiples of the fundamental frequency, but actually the transient sound of a guitar contains additional frequencies [3]. Our plan was to acquire a time record of each tone. Due to the nature of the additional analysis (see section 2.4.), the time of recording had to match an inverse value of the tone's fundamental frequency. In such a case, the spacing between frequency lines of discrete tone spectrum is equal to the fundamental frequency of the tone [6]. Thus, each frequency line of the spectrum represents the magnitude of the corresponding harmonic component of the tone and the neighboring enharmonic components. To prevent errors due to eventually non-tuned strings, each analyzed tone was recorded with a frequency resolution of approximately 1 Hz and when necessary the string was tuned to match the fundamental and higher harmonic components in an optimal way.

The time of recording, sample rate, number of samples, fundamental frequency and frequency lines spacing of the transformed signal from time to frequency domain are shown in Table 1. One can see that the frequency lines spacing almost matches the actual fundamental frequencies of the corresponding tones. Because of relatively low frequency resolution the difference between the actual fundamental frequency and frequency lines spacing logically cannot result in a significant error even if the string was not perfectly tuned (which it was, see above). Namely, the difference between the actual fundamental frequency and frequency lines spacing is negligible in comparison to the frequency resolution for all analyzed tones (see Table 1). The actual frequency components (measured) are therefore always close to the theoretical frequencies which depend on frequency lines spacing. Because of this the energy of the actual frequency components is certainly captured at the theoretical frequencies. Recording of tones started automatically after string excitation. Three different periods between string excitation and start of recording were chosen: 0.2 s, 0.6 s and 1.0 s. These time periods were based on the intensity level of the tone of an average guitar, which rapidly decreases soon after the string excitation [3]. Most tones in musical compositions last less than one second, which is another reason for the chosen time constants.

The discrete amplitude spectrum  $S'(m \cdot \Delta f)$  of the recorded tone was calculated with the Fast Fourier Transform (FFT) technique [7]:

$$S'(m \cdot \Delta f) = \frac{T}{N} \sum_{n=0}^{N-1} f(n \cdot \Delta t) \cdot e^{-j2\pi mn/N}, \quad (1)$$

where  $m = 1, 2 \dots N/2$ ,  $\Delta f$  is frequency lines spacing,  $T$  time of recording,  $N$  number of samples,  $\Delta t$  time interval between samples,  $f(n \cdot \Delta t)$  a digital value of a record at point  $n$  and  $j$  is  $\sqrt{-1}$ . One-sided amplitude spectrum  $S(m \cdot \Delta f)$  in units of Pascal is defined as [7]:

$$S(m \cdot \Delta f) = 2 \frac{|S'(m \cdot \Delta f)|}{N}. \quad (2)$$

Finally, the average amplitude spectrum  $\overline{S(m \cdot \Delta f)}$  (*i.e.*, tone spectrum) of a certain tone is calculated from 10 single spectra:

$$\overline{S(m \cdot \Delta f)} = \frac{1}{10} \sum_{i=1}^{10} S_i(m \cdot \Delta f). \quad (3)$$

Next, a tone spectrum was converted from Pascal (rms values) into dB(A) units (A-weighting, 0 dB=0.00002 Pa) [8]. Thus, a typical human response was taken into account. The first 15 frequency lines of each A-weighted tone spectrum were analyzed. For each of these frequency lines (an average of ten frequency lines) standard deviation was less than 3 dB(A).

Unfortunately, such analysis results in some kind of mixture of harmonic and enharmonic frequency components (plus adjoining noise). To establish the size of this error, we compared the timbres of good and bad tones “F”, “B” and “g”, which were recorded 0.6 s after the string excitation as described in section 2.3. The time of recording was 0.256 s, thus the frequency resolution of the transformed signal with the FFT technique was 3.90625 Hz ( $N = 4096$ ). The ratio, which expresses the relative content of the enharmonic components in the tone timbre is:

$$R = \frac{h}{h + e}, \quad (4)$$

In this expression  $h$  indicates the sum of the first 15 harmonic components in dB units and  $e$  indicates the sum of the enharmonic components in dB units in a range from 0 Hz to the frequency of the 15th harmonic component. For the bad and for the good tones  $R$  always ranged between 0.05 and 0.2, which means that the relative content of the enharmonic components in the tone timbre is not so drastic. Namely, the number of frequency lines representing the enharmonic components is at least 20 times bigger than the number of the harmonic components. For example, if  $h = 15 \times 50$  dB (15 harmonic components) and  $e = 300 \times 10$  dB (300 enharmonic components) then  $R = 0.2$ . The total sound pressure level of the 15 harmonic components is approximately 62 dB and of the 300 enharmonic components is approximately 35 dB which is an enormous difference in loudness [8]. In addition, the differences in  $R$  between the good and bad tones of the same pitch were always under 20%, which implies that “mixing” of frequency components is not such a fatal error. This is all the more true because the additional analysis (see section 2.4.) depends on the differences between the tone records rather than the tone records themselves.

### 2.3. CONDITIONS OF TONE RECORDING

A device for string excitation was designed to ensure reproducibility of experiments, because the timbre of tone depends on the direction, force, point and way of string excitation [5]. A guitar was hung on two threads, which provided good isolation from surroundings. The top plate was always perpendicular, while the excited string was parallel to the floor. The experimental set-up, microphone position and a sketch of the string excitation device are shown in Fig. 1. The weight of the string excitation device fell always from the same point (a stop was designed) and bumped into the lever of the pick. Simultaneously, on the other side of this lever, the pick excited the string. Two bars ensured the positioning of the device with respect to the string in 4 axes (see Fig. 1). Figure 2 shows the measurement set-up. The microphone position in the near field was chosen according to Prasad et al. [9], since no special characteristics (sound power, etc.) in the far field were investigated. It should be noted here that the tone recording with a different microphone position (slightly larger distance from a guitar) would also be appropriate. The reason for our choice was that the analysis depended on the differences between the tone records rather than on the tone records themselves.

## 2.4. EVALUATION OF MEASUREMENTS

A simultaneous combination of two or more tones, that is pleasing to the ear is termed consonant. When a combination of tones is not pleasing to the ear, the sound is termed dissonant. The consonant combinations of tones have a ratio of fundamental frequencies of two integer numbers none of which is large; for example, 2:1, 3:2, 5:3, 4:3, etc. [3]. The spectrum of any of the three analyzed tones can be seen as a host of tones consisting of fundamental frequency only (without higher harmonic components), which are evenly spaced. On the other hand, the same spectrum can be seen as a host of intervals where each interval consists of two frequency lines (components). Analyzing the consonance and dissonance between the fundamental frequencies of two or more simultaneously played tones is physically the same as analyzing these two or more frequency components in a certain tone spectrum. Thus we can conclude that a definition of the consonance and dissonance between the tones by Olson [3] is suitable for the explanation of consonance and dissonance inside the tone spectrum. All frequency components in this spectrum should be considered as the fundamental and only fundamental frequencies of the simultaneously played tones. For example: if we simultaneously play three tones with the fundamental frequencies of 300 Hz, 400 Hz and 600 Hz (without overtones), we hear the octave (300 Hz : 600 Hz), perfect fifth (600 Hz : 400 Hz) and perfect fourth (400 Hz : 300 Hz).

The strings of the tested guitars were tuned according to the equal temperament scale appropriate for standard guitar tuning ( $a_1=440$  Hz was used) [3]. The next definition is essential for this analysis: when the difference in Hz between a certain frequency line and fundamental frequency of some scale tone ( $a_1=440$  Hz) is less than 1%, the frequency line can be considered as a pure tone and gets the name of the scale tone. For the tone “F”, the pure tones with sample spectra of good and bad tones are shown in Fig. 3. Note that A-weighted SPL of some frequency lines (components) differ by more than 10 dB(A). The pure tones for tones “B” and “g” are shown in Table 2. The pure tones actually indicate the fundamental frequencies of the available tones on the well tuned tested guitars. Thus, when a certain frequency component of the analyzed tone spectrum is not a pure tone, it automatically increases the dissonance of this spectrum. For example: Fig. 3 shows that the 2nd frequency component coincides with the tone “F” on the scale with a  $f = 440$  Hz. The same goes for the 3rd component, which gets the name “c1”. On contrary, the difference in frequency between the 7th component (612 Hz) and the nearest available tone “d#2” (622 Hz) is more than 1%, thus this component represents the dissonance in the “F” tone spectrum. Let it be noted again that each of the 15 frequency components inside the tone spectrum is considered as a fundamental frequency of a certain tone. The effect of the 7th component (612 Hz) in a tone spectrum from Fig. 3 is similar to the effect where the first string which enables “d#2” (622 Hz - 10th fret) would be slightly out of tune according to the other strings. Despite the fact that the 14th and 7th frequency lines form a consonant interval (octave 2 : 1) inside a tone spectrum, they are considered as a dissonant interval, because the difference between them and the fundamental frequencies of any scale tone is bigger than 1% - they are not the pure tones.

The 11th and 13th frequency lines (components) were excluded from the analysis of intervals for two reasons. First, in this analysis these components represented the dissonance, but according to [3] the interval 11 : 13 is classified neither into the consonant nor into the dissonant group. Second, out of the 105 ( $15 \times 14 / 2$ ) intervals for the first 15 frequency lines, we considered 25 intervals, 19 of which are consonant. Thus, one interval more or less does not represent an important effect on the analysis. This is all the more true, because all comparisons of the tone spectra were made under the same procedure.

To sum up, 4 frequency components (7th, 11th, 13th and 14th) represent dissonant portion of the tone spectrum because of the effect of so called “non-tuned strings”. Since 15 frequency components form a harmonic series, for the rest of 11 pure tones of each tone spectrum we can use the scale of just intonation [3], which employs frequency intervals represented by the ratios of the smaller integers of the harmonic series. From the above we can conclude that this scale is a good basis for the presented consonance/dissonance definition.

The SPL of interval  $L_{ij}(k)$ , consisting of frequency lines  $i$  and  $j$ , is [8]:

$$L_{ij}(k) = 10 \cdot \log(10^{L_i(k)/10} + 10^{L_j(k)/10}), \quad (4)$$

where  $L_i(k)$  and  $L_j(k)$  are the A-weighted SPL of the first and the second frequency line of the interval  $ij$ , respectively, and  $k$  indicates one of the tones “F”, “B” or “g”. The 25 considered combinations of  $ij$  are shown in Table 3: the classification into the consonant and dissonant intervals was made according

to Fig. 4, which shows the consonance-dissonance characteristics for various interval ratios [3]. The limit between consonance and dissonance is represented by the *order of merit* = 6.

The difference of  $L_{ij}(k)$  for the good and bad tones of the same pitch yields the parameter  $\Delta L_{ij}(k)$ :

$$\Delta L_{ij}(k) = L_{ij}(k: \text{good guitar}) - L_{ij}(k: \text{bad guitar}). \quad (5)$$

The following terms can now be defined:

- The consonance ( $C_G(k)$ ) of a good tone  $k$  relative to a bad tone  $k$ , is defined as the sum of the consonant  $\Delta L_{ij}(k)$  that are larger than 0.
- The dissonance ( $D_G(k)$ ) of a good tone  $k$  relative to a bad tone  $k$ , is defined as the sum of the dissonant  $\Delta L_{ij}(k)$  that are larger than 0.
- The consonance ( $C_B(k)$ ) of a bad tone  $k$  relative to a good tone  $k$ , is defined as the sum of the consonant  $\Delta L_{ij}(k)$  that are smaller than 0.
- The dissonance ( $D_B(k)$ ) of a bad tone  $k$  relative to a good tone  $k$ , is defined as the sum of the dissonant  $\Delta L_{ij}(k)$  that are smaller than 0.

Consonance and dissonance of sample good and bad tones are shown in Table 4.

### 3. RESULTS AND DISCUSSION

Since each tone of bad and good guitars was recorded in three different periods after string excitation, 144 comparisons between good and bad tones were made, *i.e.*:

- each good tone "F" with each bad tone "F" (four good guitars, four bad guitars and three different periods after string excitation results in 48 comparisons),
- each good tone "B" with each bad tone "B" (48 comparisons),
- each good tone "g" with each bad tone "g" (48 comparisons).

For all comparisons, the following expression applied:

$$(C_G(k) + C_B(k)) > 2 \cdot (D_G(k) + D_B(k)) \quad (6)$$

This expression is named the 'rule of consonance-dissonance' (RC-D) and provides an objective criterion for distinguishing an extremely bad guitar tone from a good one. The difference between the quality of bad and good tested tones was drastic (see section 2.1.), thus the 'rule of consonance-dissonance' is too rough for being applied without any modifications. We can see from expression (6) that the condition "is greater" will be fulfilled more easily when:  $C_G$  is as large as possible,  $|C_B|$  is as small as possible,  $D_G$  is as small as possible and  $|D_B|$  is as large as possible (mind that  $C_B$  and  $D_B$  are negative in the above definitions). According to this, an expression which indicates the relative quality of any guitar tone (presumably bad) in comparison to the presumably good guitar tone is:

$$Q(k) = D_G(k) + D_B(k) - C_B(k) - C_G(k). \quad (7)$$

If the presumably bad tone is really worse than the presumably good tone, then  $Q(k)$  is negative. Larger  $Q(k)$  indicates a better quality of the bad (*i.e.*, tested) tone in comparison to the good tone. We can see from eq (7) that in the case of positive  $Q(k)$ , the tested tone (presumably bad) is actually better than the presumably good tone, therefore the assumption about the relative quality of the two tones was wrong.

The RC-D is based on some physical and musical facts (see section 2.4.), thus we assume that it applies to all guitar tones, not only to the three measured tones. To confirm that the measured differences in the tone spectra of good and bad guitars are explainable also by frequency response function (*i.e.*, FRF) of a guitar, the following experiment was performed. Excitation of a guitar was performed by a mechanical impulse at the bridge and the response signal was a sound pressure at 1 m from the guitar [10]. Figure 5 shows a typical difference between FRFs corresponding to a bad and good guitar (see section 2 for guitar quality ratings). The first resonant peak in FRF is a result of interaction of the *top board-air-back board* triplet [11], these components being in fact the most important parts of guitar body [2]. In addition, this peak corresponds to a normal mode since for the resonant frequency response signal lags 90 degrees with respect to the input signal (not shown). One

can see that the amplitude of the first resonant peak in FRF is significantly higher and its damping is lower for the good guitar in comparison to the bad one. We can conclude with high certainty that under the given circumstances (the conditions of testing [10]) the first resonant peak in FRF of a guitar is an appropriate criterion for its tonal quality. This is reasonable because a relatively large amplitude and relatively low damping of this peak indicate a relatively strong and low damped acoustic response which is a feature of a good instrument. The performed experiment highlighted also the statement about importance of low-order modes (see section 1). It seems that the lowest radiating mode of a guitar (1st resonant peak in FRF) is important for its acoustic characteristics in all registers: Both higher and lower tones corresponding to the good guitar(s) were preferred in comparison to the tones of the bad guitar(s) (see section 2). An exact and physical explanation of a correlation between the FRF of a guitar and its tones is difficult [12, 13] and the most probable reason for this is a complicated modal behavior of an instrument [14].

Because the RC-D is based on the interval analysis, this is effective only when each of the considered 15 frequency components has some energy. If not so, we cannot speak about "loudness of interval", but only about "loudness of pure frequency". To ensure such conditions, the recording of tones was performed close to the guitars, where each of the first 15 frequency components had a perceptible portion. Finally, an average quality of the three tested tones "F", "B" and "g" is:

$$Q_m = (Q("F") + Q("B") + Q("g"))/3. \quad (8)$$

Any guitar whose quality is compared to a high quality guitar can be seen as a tested guitar with tested tones. Thus,  $Q_m$  actually indicates the average tone quality of the tested guitar. This is based on our experiences that good and bad guitars have pretty equal quality of all tones in practice. We never encountered a guitar with one or more tones significantly alighted from other tones. Anyway, it is reasonable to calculate  $Q_m$  for more than three tones, and they should represent the whole tone register of the guitar.

#### 4. CONCLUSION

Objective determination of tone quality of a musical instrument is an important problem [15, 16]. The proposed criterion for the relative quality of the guitar tone is a result of objective sound measurements of bad and good classical guitars. The criterion, the 'rule of consonance-dissonance' (RC-D), was expressed in a mathematical form (see eq (6)) and interpreted in terms of the physical and musical theory (see section 2.4.). The essential difference between the bad and good timbre of a guitar tone is in the relative contributions of the consonant and dissonant combinations of frequency lines in the discrete amplitude spectrum of the tone. The described criterion includes not only the analysis of loudness but also the quality analysis of tones. We believe that the RC-D could be applied to all guitar tones, since RC-D allows a simple comparison of any two guitar tones of the same pitch (recorded under conditions described in sections 2.2. and 2.3.). Secondly, RC-D enables a systematic pursuing of tone quality during guitar production, and should also facilitate improvements of the instrument after its assembly.

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TABLE 1

Properties of recorded tones

	tone "F"	tone "B"	tone "g"
Time of recording (s) - $T$	0.011609	0.008	0.005
Sampling frequency (kHz) - $f_S$	44.1	32.0	25.6
Number of samples - $N$	512	256	128
Actual fundamental frequency (Hz) - $f_F$	87.3	123.5	196.0
Freq. lines spacing after frequency transformation (Hz) - $\Delta f$	86.13	125.0	200.0

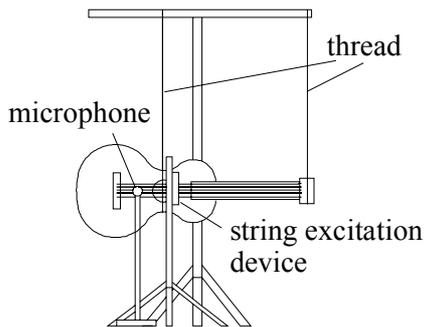
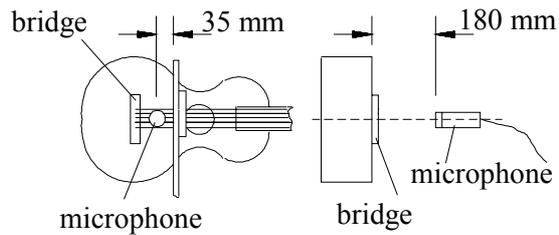
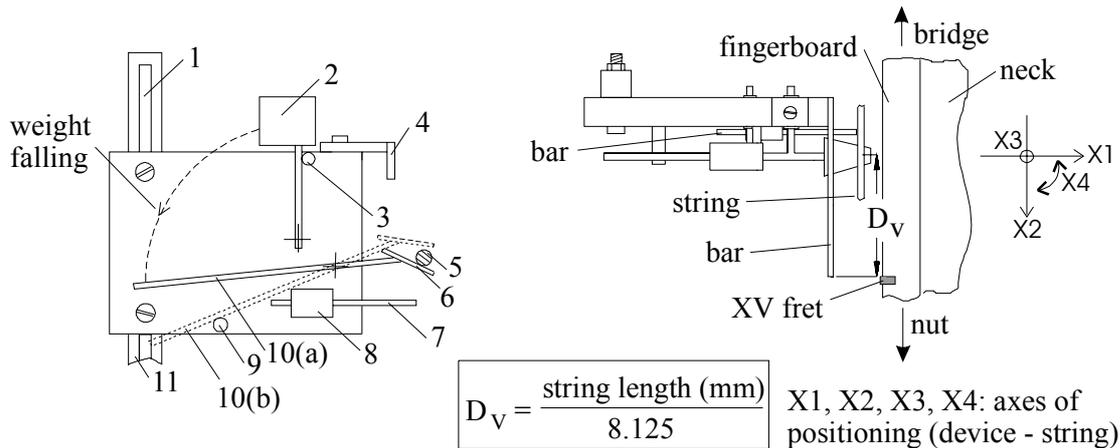
**EXPERIMENTAL SET:****MICROPHONE POSITION:****STRING EXCITATION DEVICE:**

Figure 1: Guitar, microphone and the string excitation device. 1, the slot for adjusting the vertical position of the device. 2, weight. 3, stop for the weight. 4, the bar for adjusting the string excitation device parallel to the string. 5, string. 6, pick. 7, the bar for adjusting the distance between the string excitation device and the string. 8, guide for a bar. 9, stop for the lever of the pick. 10, lever of the pick [(a) before the string excitation, (b) after the string excitation]. 11, stand.

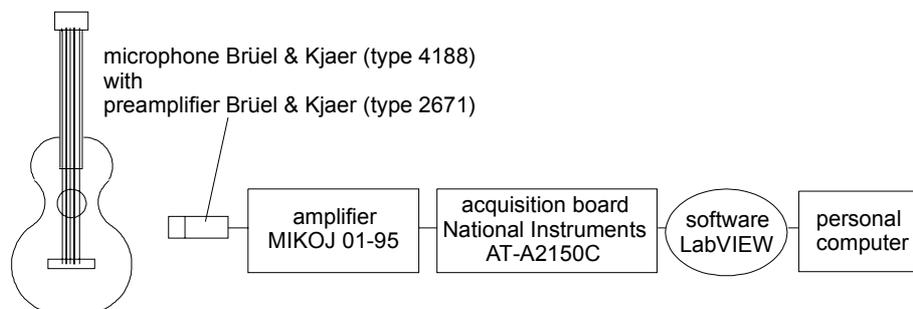


Figure 2: Measurement set-up.

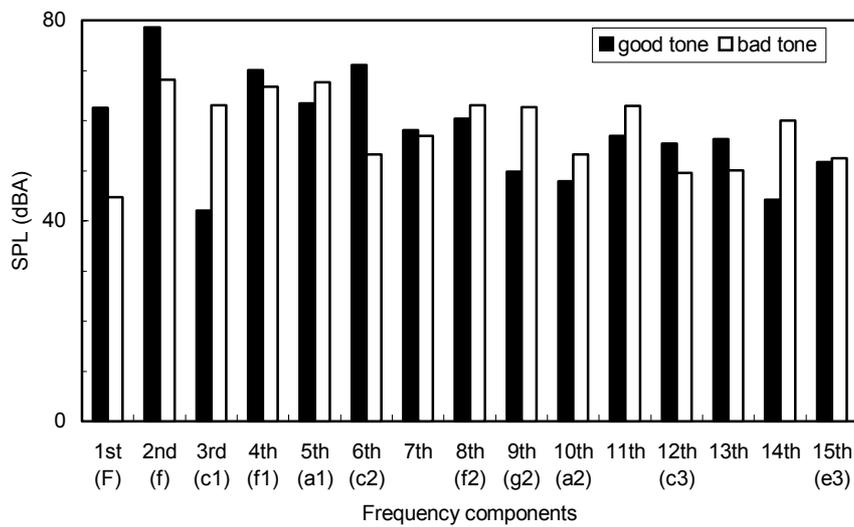


Figure 3: Good and bad timbre of tone "F" recorded 0.2 s after string excitation. The names of the pure tones are in parentheses under the numbers of the corresponding frequency lines (components).

TABLE 2

The pure tones for tones "B" and "g"

frequency line	1st	2nd	3rd	4th	5th	6th	7th	8th
pure tone (for "B")	B	b	f <sup>#</sup> 1	b1	d <sup>#</sup> 2	f <sup>#</sup> 2	/	b2
pure tone (for "g")	g	g1	d2	g2	b2	d3	/	g3
frequency line	9th	10th	11th	12th	13th	14th	15th	
pure tone (for "B")	c <sup>#</sup> 3	d <sup>#</sup> 3	/	f <sup>#</sup> 3	/	/	a <sup>#</sup> 3	
pure tone (for "g")	a3	b3	/	d4	/	/	f <sup>#</sup> 4	

<sup>#</sup> indicates pitch increase for a semitone

TABLE 3

Considered intervals: combinations of  $ij$ .

	consonant			consonant			consonant			dissonant	
	$i$	$j$		$i$	$j$		$i$	$j$		$i$	$j$
<i>octave</i>	2	1	<i>perfect fifth</i>	6	4	<i>major third</i>	10	8	<i>minor sixth</i>	8	5
<i>octave</i>	4	2	<i>perfect fifth</i>	9	6	<i>major third</i>	15	12	<i>minor third</i>	6	5
<i>octave</i>	6	3	<i>perfect fifth</i>	12	8	<i>perfect fourth</i>	4	3	<i>minor seventh</i>	9	5
<i>octave</i>	8	4	<i>major sixth</i>	5	3	<i>perfect fourth</i>	8	6	<i>major tone</i>	9	8
<i>octave</i>	10	5	<i>major sixth</i>	10	6	<i>perfect fourth</i>	12	9	<i>major seventh</i>	15	8
<i>octave</i>	12	6	<i>major sixth</i>	15	9				"octave"	14	7
<i>perfect fifth</i>	3	2	<i>major third</i>	5	4						

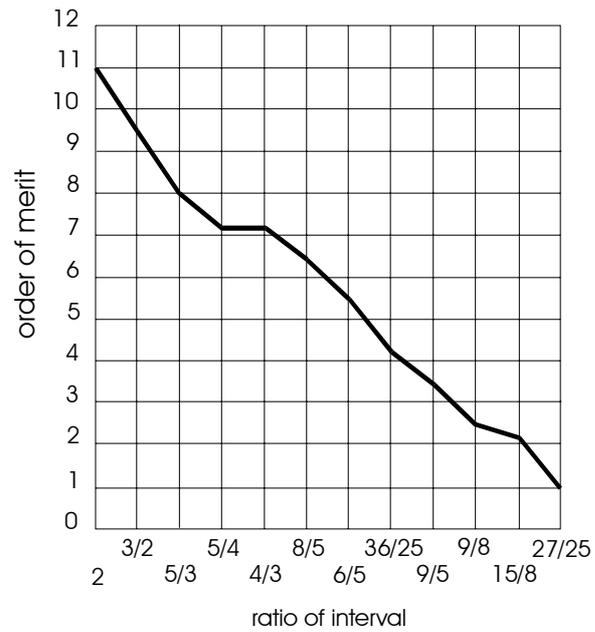


Figure 4: Consonance-dissonance characteristic for various interval ratios [3]

TABLE 4

Consonance and dissonance of good and bad tones

	"F"			"B"			"g"		
	0.2 s	0.6 s	1.0 s	0.2 s	0.6 s	1.0 s	0.2 s	0.6 s	1.0 s
$C_G(k)$ ; dBA	97.5	99.0	123.7	159.7	174.4	198.3	62.2	45.3	13.2
$D_G(k)$ ; dBA	3.9	7.6	13.0	8.2	7.4	1.8	16.3	15.8	0.0
$C_B(k)$ ; dBA	-30.1	-22.5	-16.8	-8.9	-3.5	-2.4	-6.2	-4.3	-15.7
$D_B(k)$ ; dBA	-20.2	-23.6	-23.6	-10.8	-10.1	-23.9	-1.2	-6.4	-7.3

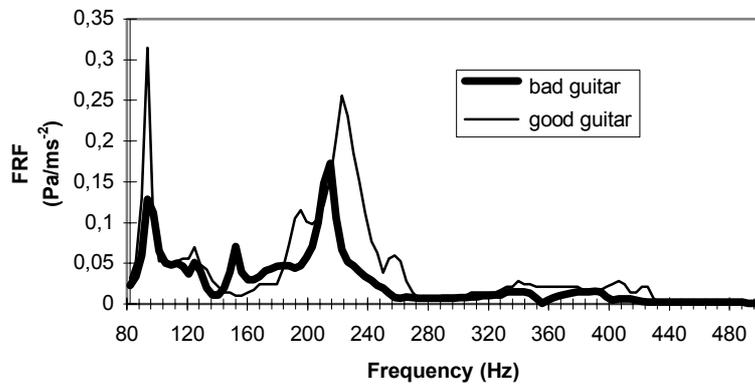


Figure 5: FRF for a bad and good guitar.